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EVALUATION OF A BURST-ERROR CORRECTION CODE ON A GILBERT CHANNEL

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FOREWORD

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Thanks are also due to Mrs. Helen Kwasniewski who converted the author's many problems into a form suitable for computer evaluation.

AIR FORCE, Canterbury Press, Rome, N.Y., 6/15/62, 187

ABSTRACT

Error correction codes have been designed to allow the communication engineer to add a degree of error protection to transmitted digital data. Difficulties arise, however, when it is desired to obtain maximum efficiency from the redundant digits added to a particular channel. This report illustrates a technique for evaluating the improvement afforded a fading-type channel by the use of a burst-error correcting code. The Gilbert model of a fading channel is then evaluated using this technique. A series of curves are presented showing the improvement in error rate afforded this particular channel by a burst-error correction code as a function of the reduced information rate.

PUBLICATION REVIEW

This report has been reviewed and is approved.

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RADC-TDR-62-189

EVALUATION OF A BURST-ERROR CORRECTION CODE ON A GILBERT CHANNEL

INTRODUCTION

Several means are available to the communications engineer to increase the probability of correctly receiving digital information transmitted over a noisy channel. Shannon[1] proved that by properly encoding the information it is theoretically possible to communicate over a noisy channel with an arbitrarily small probability of error, if the transmission rate is less than the channel capacity. Since then much effort has been concentrated on developing coding techniques that will approach this theoretical limit with emphasis on codes which can be easily implemented.

The error correction capability of codes generally fall into one of two basic classes: random error correction codes and burst-error correction codes. A random error correction code will correct errors that occur randomly within a block of digits, while a burst-error correction code will correct errors that are grouped over a span of digits. In both classes the error correction capability is obtained by adding redundancy to the information by means of some mathematical algorithm. How much protection can be obtained by adding redundancy, i.e., reducing the information rate, is somewhat obscure.

One would suspect that burst-error correction codes would be more efficient than random error correction codes for transmission over many digital channels presently in use. This results from the observation that many digital channels possess a certain degree of "memory" so that when errors do occur, they tend to be grouped over a span or burst of digits. By inferring that errors of this type can be more efficiently corrected by a burst-error correction code, it is meant that a given degree of error protection can be provided for a smaller reduction in information transmission rate.

Corrington and Ausley[2] have performed an analysis that shows the decrease in probability of error on a binary symmetric channel when it is protected by a random error correction code. It is the purpose of this report to show the results of a similar analysis performed on a fading channel protected by a burst-error correction code. Curves, that have been analytically obtained, are presented showing the amount of error protection provided by a burst-error correction code as a function of the reduced information rate for given noise conditions.

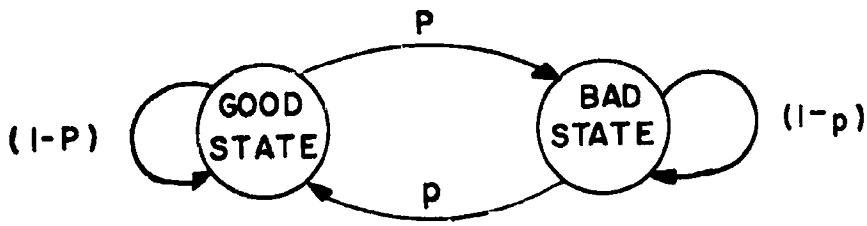
There are two types of burst-error correction codes: the recurrent codes, such as the codes designed by Hagelbarger[3] and Kilmer[4], and the block codes, such as those designed by Elspas.[5] Certain block codes, in particular the shift register-generated cyclic codes developed by Elspas, are capable of correcting a burst of digits at the end of the block as well as a burst of digits at the beginning of the block, provided the sum of the digits in both bursts does not exceed the burst correction capability of the code. Error patterns of this type are called a closed-loop burst of errors. If a burst-error correction code cannot correct this particular error pattern, it is called an open-loop burst-error correction code.

This report will show the gains achievable through the use of open-loop burst-error correction codes of the block type. Several codes are considered with block lengths varying between 7 and 127 with burst-error correction capabilities varying between 1 and 56.

This report is intended to supplement the first interim report from Stanford Research Institute under RADC Contract AF30(602)-2327, entitled, "Design and Instrumentation of Error Correction Codes, " by Bernard Elspas. In this report Elspas derives the equation for the probability of block error for a wide class of channels where errors occur in bursts. This equation is used as a basis for the work to follow.

MATHEMATICAL MODEL

The mathematical model of the fading channel used for all calculations described in this paper is the model derived by Gilbert.[6] This model describes the channel as a two-state Markov process, literally a "good" state and a "bad" state. When the channel is in the good state, no errors occur (the probability of error is equal to zero). When in the bad state, errors occur with finite probability $(1 - h)$. The Gilbert channel is illustrated below.



P is the probability of transfer from the good to the bad state and p is the probability of transfer from the bad to the good state. The probabilities, P and p, are called transition probabilities.

This model by no means characterizes all burst-error channels, but a reasonable description of a fading channel can be obtained by strategically assigning values to h, P, and p. Each curve shown in this report is plotted for constant values of P and p. The probability of digit error in the bad state $(1 - h)$ has been set at .5 and kept constant for all calculations shown in this report. This represents the worst possible case since h must lie in the range, $0 \leq (1 - h) \leq .5$. This assumption is made to reduce the amount of computation and does not bias the data in favor of the error correction capability of the code.

Let $u(k)$ be the probability of a run of *at least* k correct digits following the occurrence of an error. Gilbert has shown that, $u(k) = (Q + hq)\, u\,(k - 1) + h(p - Q)\, u\,(k - 2)$ where $Q = 1 - P$; $q = 1 - p$. The initial conditions are: $u(0) = 1$ $\quad u(1) = p + hq$. Let $v(k)$ be the probability of a run of *exactly* k correct digits following the occurrence of an error. $v(k)$ satisfies the same recursion relationship as $u(k)$, but with initial conditions, $v(0) = (1 - h)q$, $v(1) = (1 - h)\,(pP + hq^2)$.

BURST-ERROR CORRECTION EFFICIENCY

An open-loop burst b error correcting code is defined as a code capable of correcting all possible combinations of errors that occur over a width of less than or equal to b digits. The width of a burst is defined as the number of digits from the first digit in error to the last digit in error inclusive within the message block. A burst b error correcting code can correct all errors that occur within a span of digits $w \leq b$.

Elspas[8] has shown that the probability $(1 - P'e)$ that an n digit message will be correctly received when it is protected by a burst b error correcting code after transmission through a channel described by the Gilbert model is:

$$(1 - P'e) = P(1) \left[\sum_{w=1}^{b} \sigma(w) \sum_{i=w}^{n} u(i-w)u(n-i) + \sum_{i=n}^{\infty} u(i) \right] . (1)$$

$P(1)$ is the long term probability of digit error. For the Gilbert channel $P(1)$ is defined by the equation:

$$P(1) = \frac{1}{\sum_{k=0}^{\infty} u(k)} = \frac{(1-h)P}{(P+p)} .$$

$\sigma(w)$ is a polynomial function of the $v(i)$'s determined from the possible burst patterns, and is determined from the equation:

$$\sigma(w+1) = v(0)\,\sigma(w) + v(1)\,\sigma(w-1) + \ldots + v(w-1)\,\sigma(1) .$$

By definition $\sigma(1) = 1$ and $\sigma(2) = v(0)$.

The formula for $(1 - P'e)$ can be made more tractable by eliminating the infinite summation in the following manner.

Since: $$\sum_{i=0}^{\infty} u(i) = \frac{1}{P(1)}$$

then $$\sum_{i=n}^{\infty} u(i) = \frac{1}{P(1)} - \sum_{i=0}^{n-1} u(i)$$

$$(1 - P'e) = 1 + P(1) \left[\sum_{w=1}^{b} \sigma(w) \sum_{i=w}^{n} u(i-w)u(n-i) - \sum_{i=0}^{n-1} u(i) \right] . (2)$$

We are concerned with obtaining an expression for the reduced information rate due to the addition of redundant digits. Campopiano[7] has derived an expression for the maximum number of redundant digits, r, required for a burst b error correcting code of a given block length n. This bound is (for binary codes),

$$r_{max} = 2(b-1) + \log_2 \left[n - 2(b-1) \right].$$

If there are k information digits then $r = n-k$ and the reduced information rate is k/n. We can now solve for the reduced information rate in terms of the Campopiano bound.

$$\frac{k}{n} = 1 - \frac{2(b-1) + \log_2 \left[n - 2(b-1) \right]}{n} \tag{3}$$

for $b \leq 2$.

$$\frac{k}{n} = 1 \quad \text{for} \quad b = 0. \tag{4}$$

$$\frac{k}{n} = 1 - \frac{\log_2 \, (n+1)}{n} \qquad \text{for} \quad b = 1 \; . \tag{5}$$

This bound states that a burst error correction code, using this many redundant digits, can always be found. In some cases, more efficient codes exist that will correct a longer burst for a given redundancy than the Campopiano bound would indicate. The Campopiano bound was used in this report in order to assure the existence of a burst-error correction code for every point shown on the curves.

The expressions for both k/n and $(1-P'e)$ have been developed for open-loop burst-error correction codes. No credit then is given to codes capable of correcting a closed-loop burst of errors.

Digit blocks of length (2^x-1) were chosen with x an integer in the range $3 \leq x \leq 8$. For these block lengths, b was varied in the range $0 \leq b \leq \frac{n-1}{2}$, with b restricted to be an integer. $(1-P'e)$ and k/n were calculated for each value of b and n for the given Gilbert channel conditions.

It is interesting to note at this point that the artifice of the Gilbert channel was used only for the calculations of $P(1)$, $u(k)$ and $v(k)$. Equations 1 and 3 through 5 are more general and can be used for any channel for which these statistics can be obtained, provided only that the lengths of the runs of error free digits are statistically independent. Gilbert has indicated whenever this independence can be assumed, a more elaborate model might use the distribution of $v(k)$'s directly as parameters.

DATA ANALYSIS

The data shown in this report were computed on a Burroughs 205 Digital Computer and represent the solution of the equation developed by Elspas for the probability that a coded binary digit block of length n will be error free when protected by a burst b error correction code. These values were plotted for various values of k/n. (Figures 1 - 25.)

Examination of the curves would lead to some unusual conclusions. Note that on most of the graphs the curves have obvious cross-over points. In order to make this clearer, several of the graphs have been plotted logarithmically in the range of probabilities between .90 and 1.0. These logarithmic plots are shown as insets on the main graph. The fact that these curves have cross-over points would seem to indicate that the probability of correctly receiving a large block of digits is greater than the probability of correctly receiving a smaller block of digits for the same reduction in information rate and for the same noise conditions. In particular, Figure 4 would indicate that the probability of correctly receiving a block of 127 digits is greater than the probability of correctly receiving a block of 63 digits when the reduced information rate lies in the range $.49 \leq k/n \leq .88$. Note also on this same graph that more blocks of 127 digits will be received error free than blocks of 7 digits for $k/n = .75$. Similar examples are noted on other graphs. It is well-known that greater error correction efficiency can be obtained by transmitting the coded information in longer blocks. This improvement is shown by Corrington and Ausley for optimum decoding of random errors.[3] It is not unreasonable to expect to see a similar improvement for burst-error correction codes used on a fading channel.

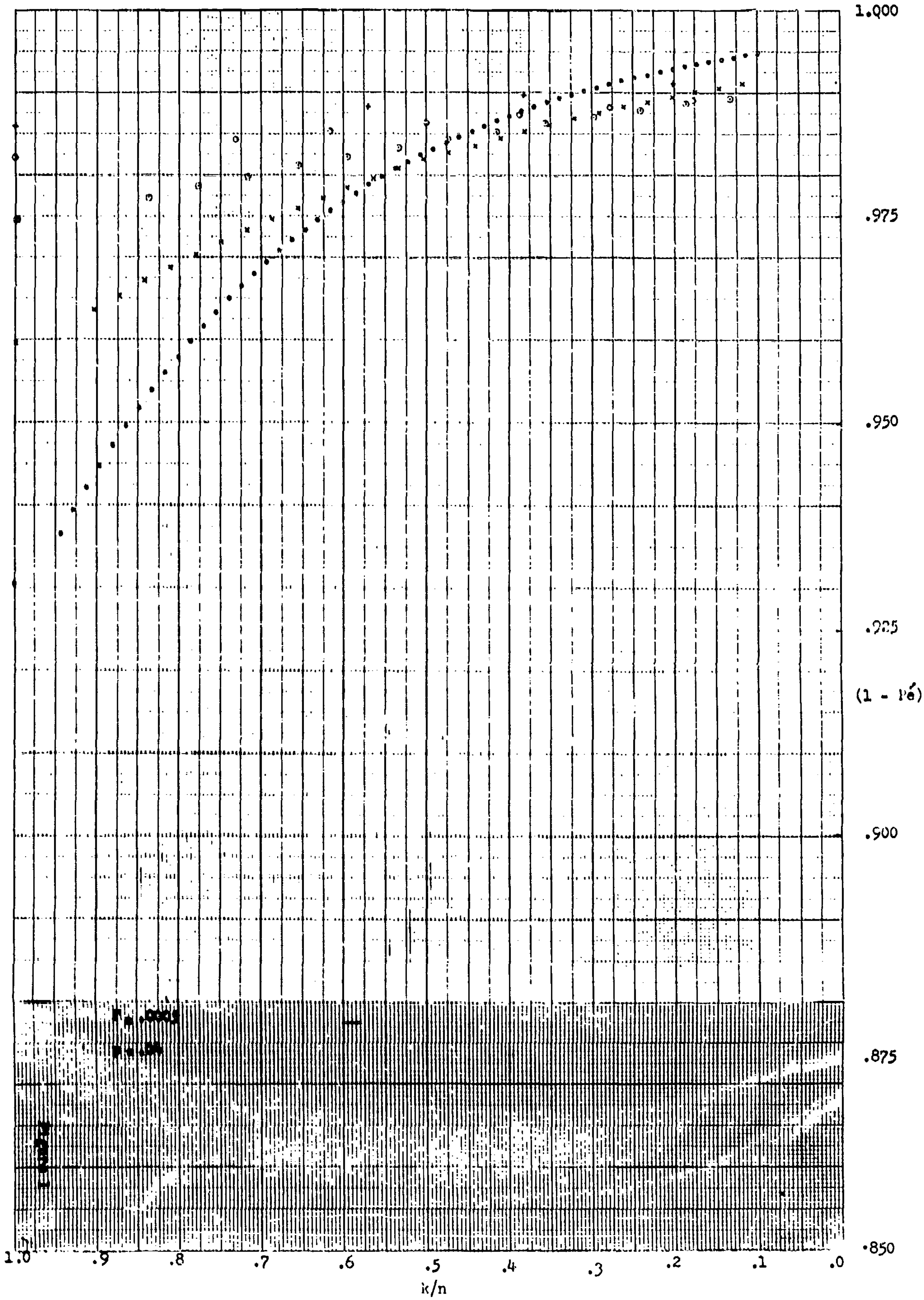
The cross-over points do not occur in the same position on all the graphs but vary as the noise conditions are changed. This would lead to the thought that there exists an optimum block length for a single burst-error correction code for given noise conditions and information rates.

Great care must be taken in stating that a given block length is better or worse than a block of a different length. Something can be said, however, when a longer block is compared with a shorter one. Let $(1-P'e)_{n_1}$ be the probability that a block of length n_1 is correctly received. Then block length n_1 is better than block length n_2 if $(1-P'e)_{n_1} \geq (1-P'e)_{n_2}$ and $n_1 > n_2$. If, however, the first inequality were true $n_1 < n_2$, no information could be gained as to which of the two block lengths would be the better.

CONCLUSION

A large number of curves representing the improvement in message error rate obtained through the use of burst-error correction codes on a Gilbert channel are presented. Since many channels may be approximated to some degree by the Gilbert channel, these curves may be useful in giving the communication engineer a priori knowledge of the received information error rate.

Although these curves apply only for the Gilbert channel, the technique used is valid for a more general channel. A similar analysis can be performed on a particular channel by gathering sufficient data to statistically approximate the parameters $u(k)$, $v(k)$ and P_1, provided the distribution of $v(k)$'s is statistically independent.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.975
.950
.925
(1 - Pé)
.900
.875
.850
1.0
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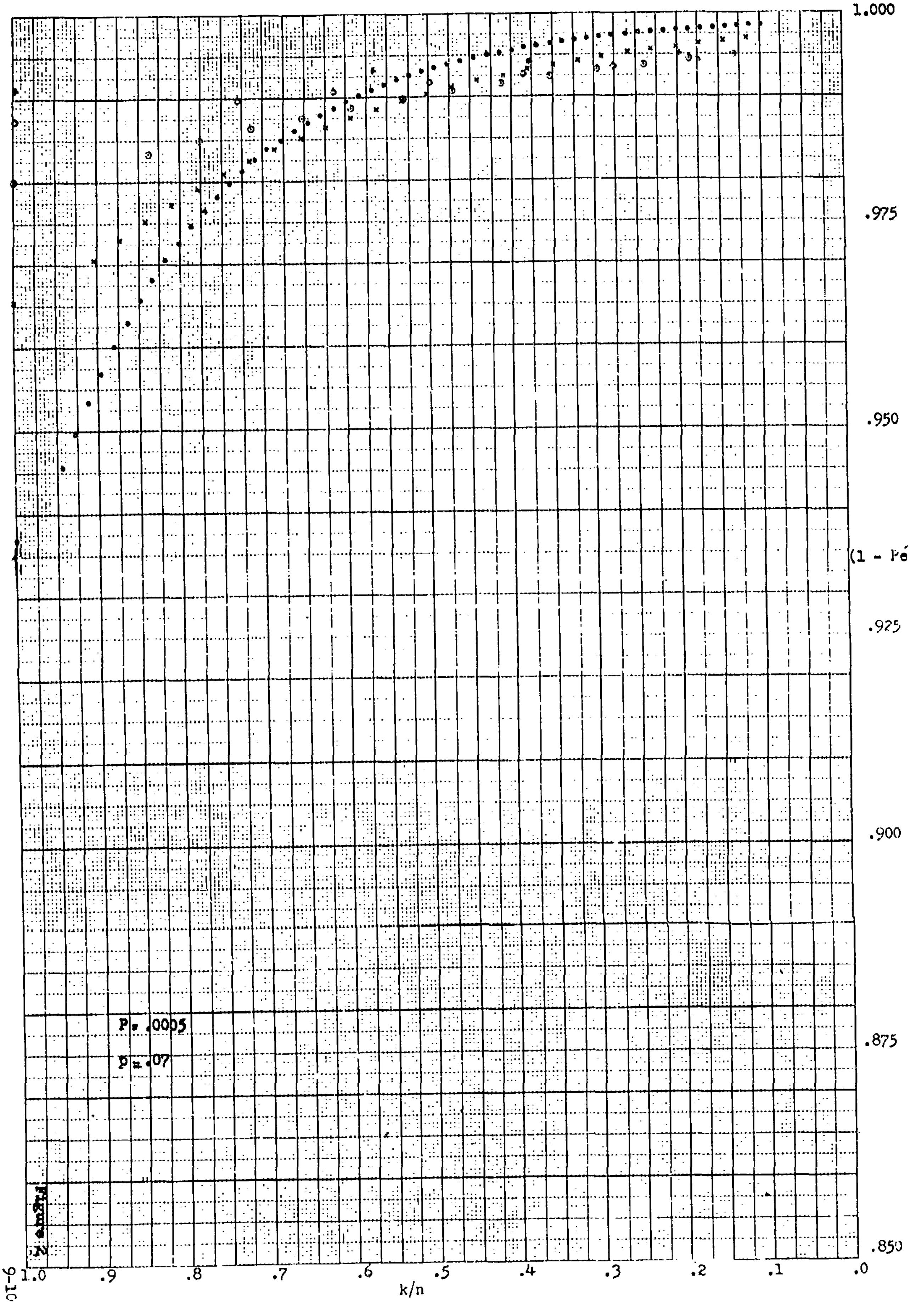
.7
.6
.5
.4
.3
.2
.1
.0
k/n


1.000
.975
.950
(1 - Pe
.925
.900
.875
.850
P = .0005
p = .07
Figure 2
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

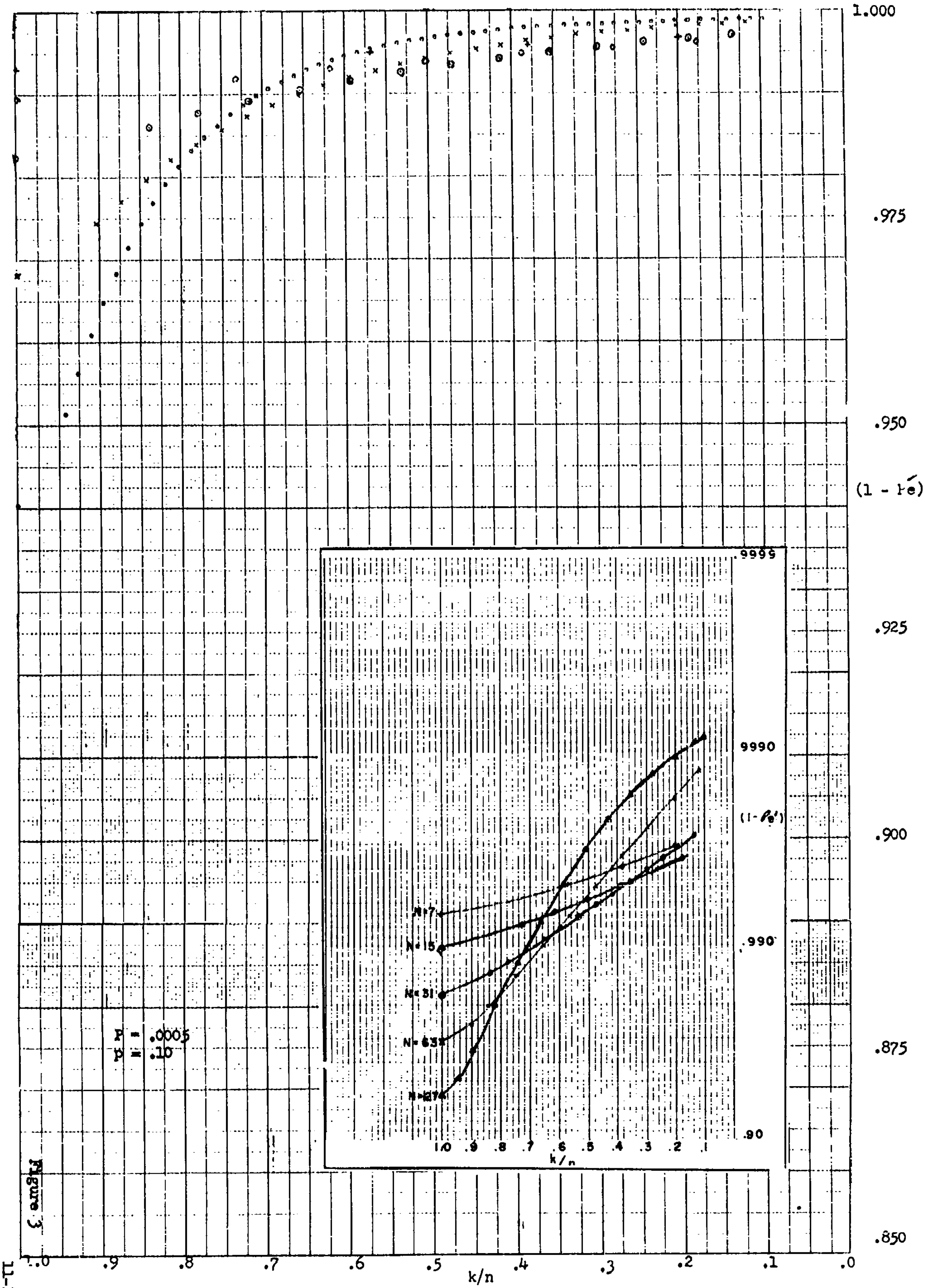

1.000
.975
.950
(1 - Pe')
.925
.900
.875
.850
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
P = .0005
p = .10
.9995
9990
(1-Pe')
.990
.90
N=7
N=15
N=31
N=63
N=127
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
k/n

Figure 3

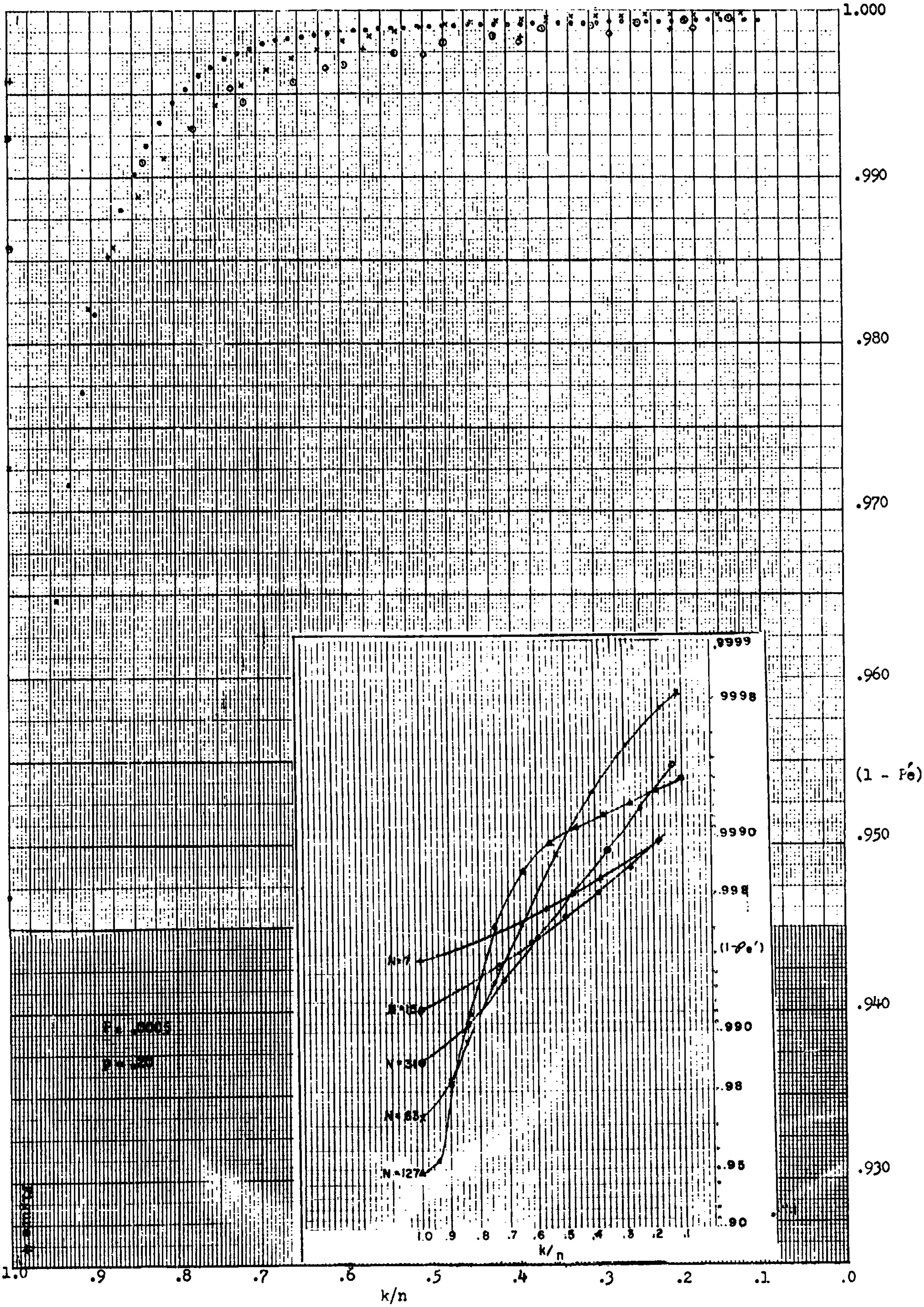
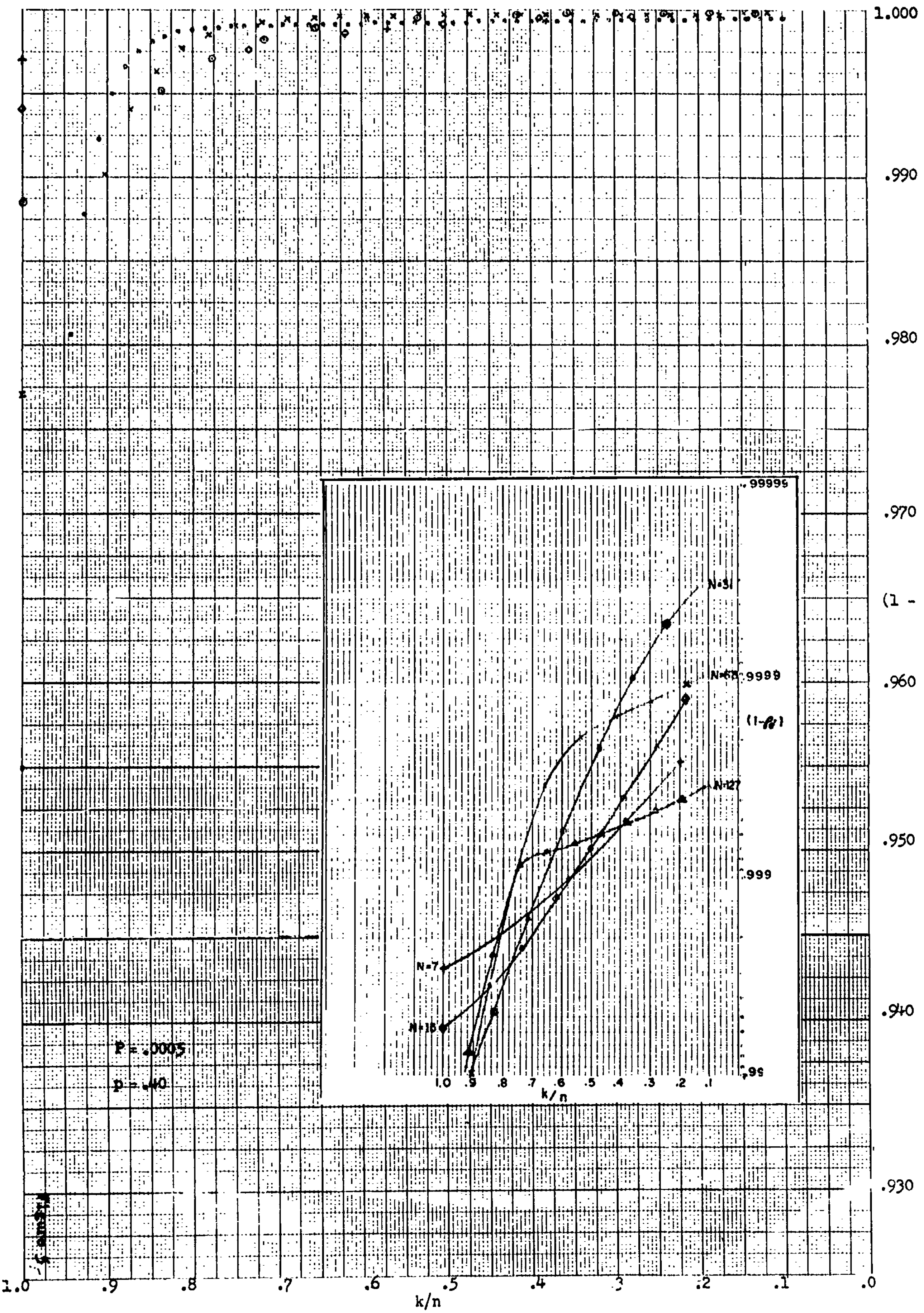

1.000
.990
.980
.970
.960
.950
.940
.930
(1 - Pé)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
P = .0005
.9999
.9998
.9990
.998
(1-Pe')
.990
.98
.95
.90
N=7
N=15
N=31
N=63
N=127
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
k/n

Figure 4


1.000
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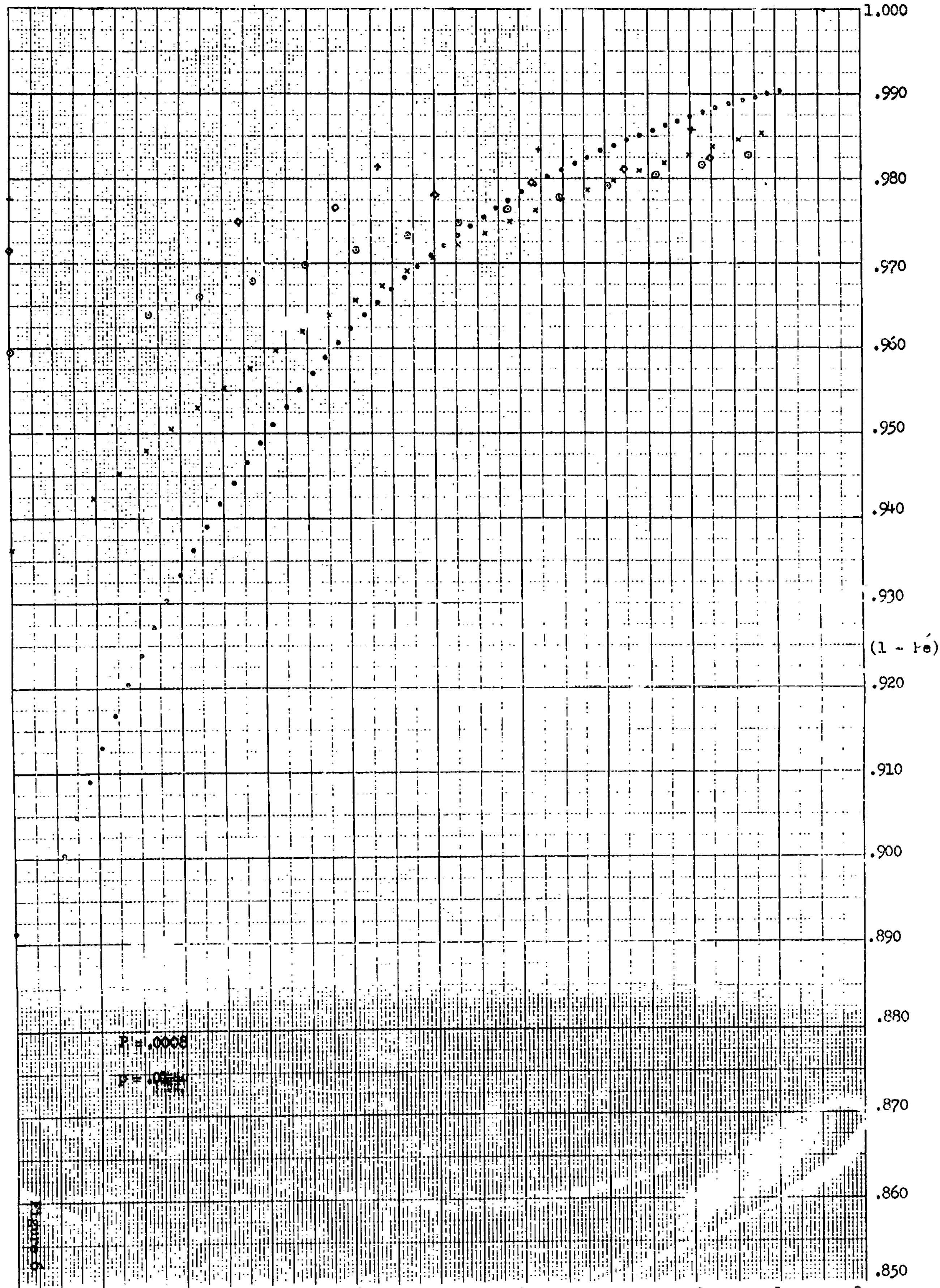
.980
.970
(1 -
.960
.950
.940
.930
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
P = .0005
p = .40
.99999
.9999
.999
.99
(1-β)
N=7
N=15
N=31
N=63
N=127
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
k/n

Figure 4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.990
.980
.970
.960
.950
.940
.930
(1 - Pe')
.920
.910
.900
.890
.880
.870
.860
.850
P = .0008
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 6

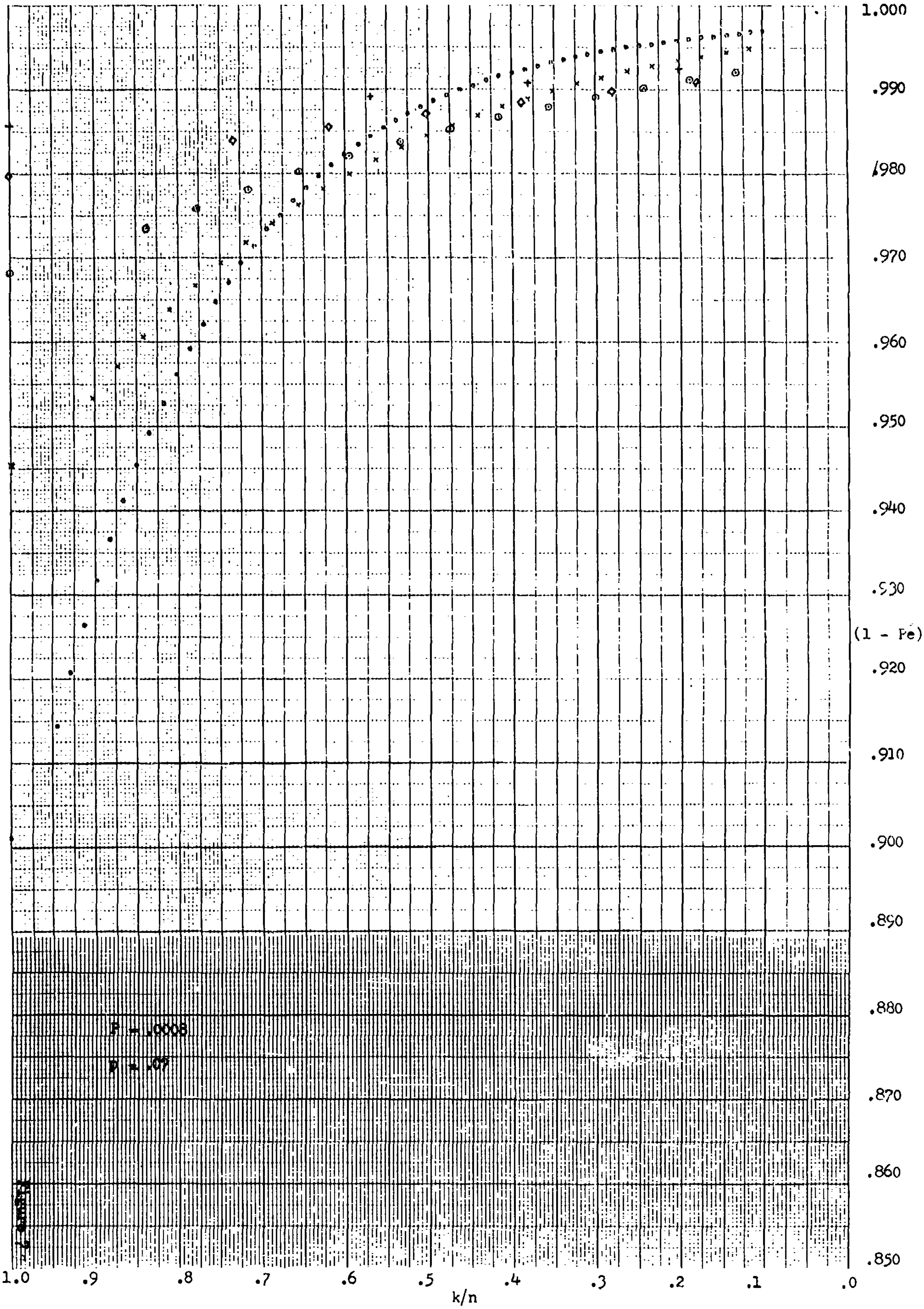
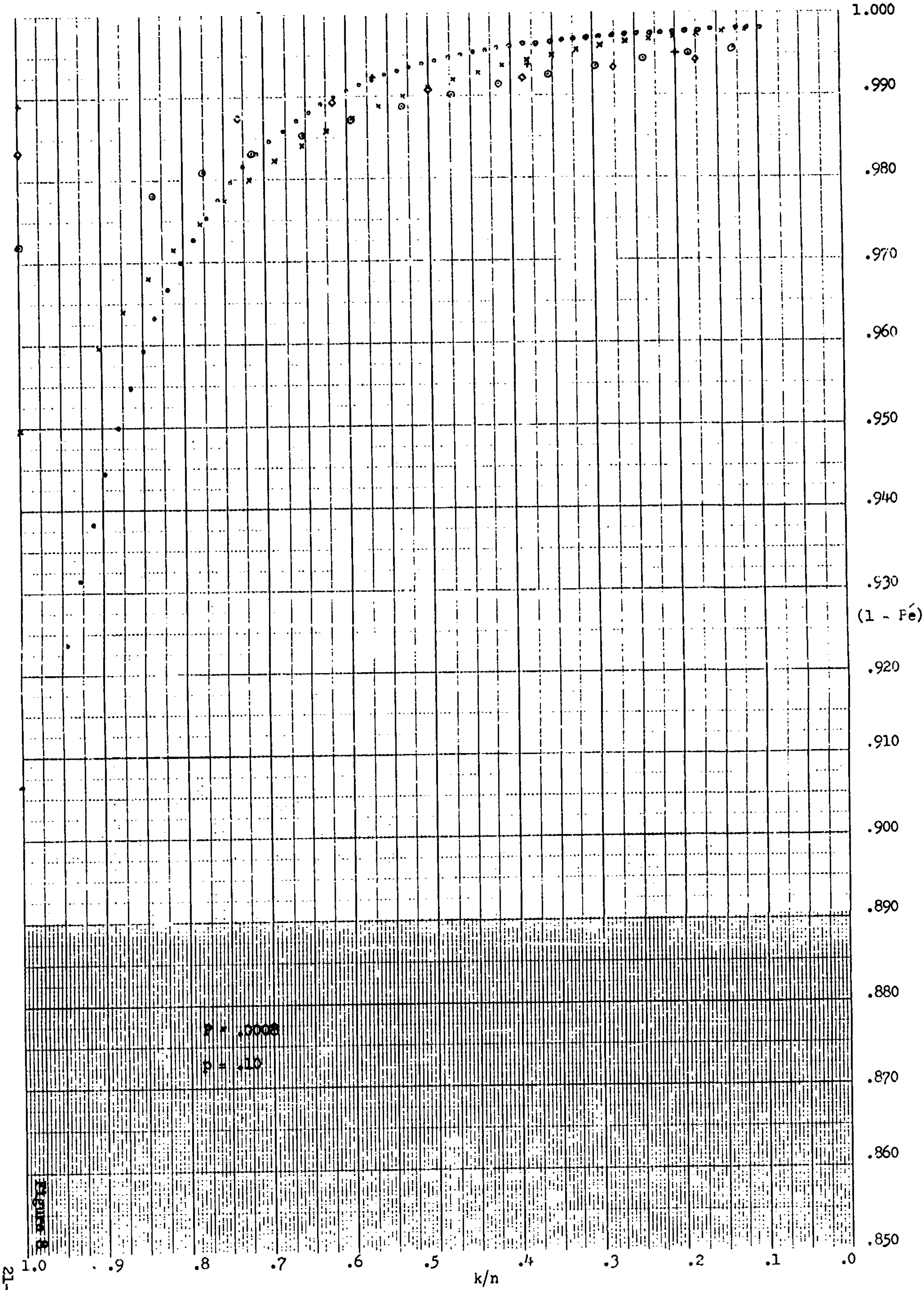

1.000
.990
.980
.970
.960
.950
.940
.930
(1 - Pe)
.920
.910
.900
.890
.880
.870
.860
.850
P = .0008
p = .07
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 7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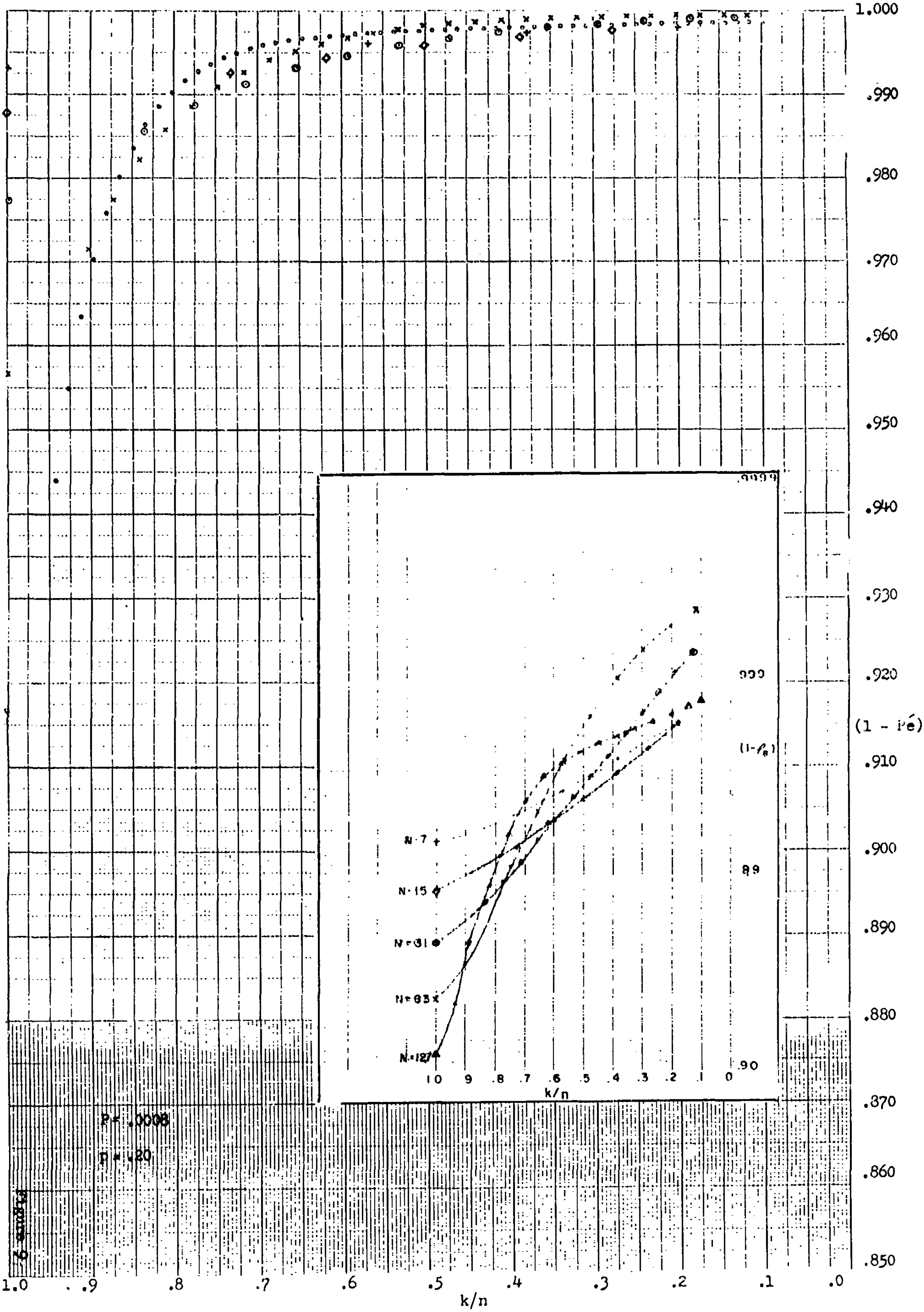
.980
.970
.960
.950
.940
.930
(1 - Pé)
.920
.910
.900
.890
.880
.870
.860
.850
P = .0008
p = .10
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 8



1.000
.990
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.960
.950
.940
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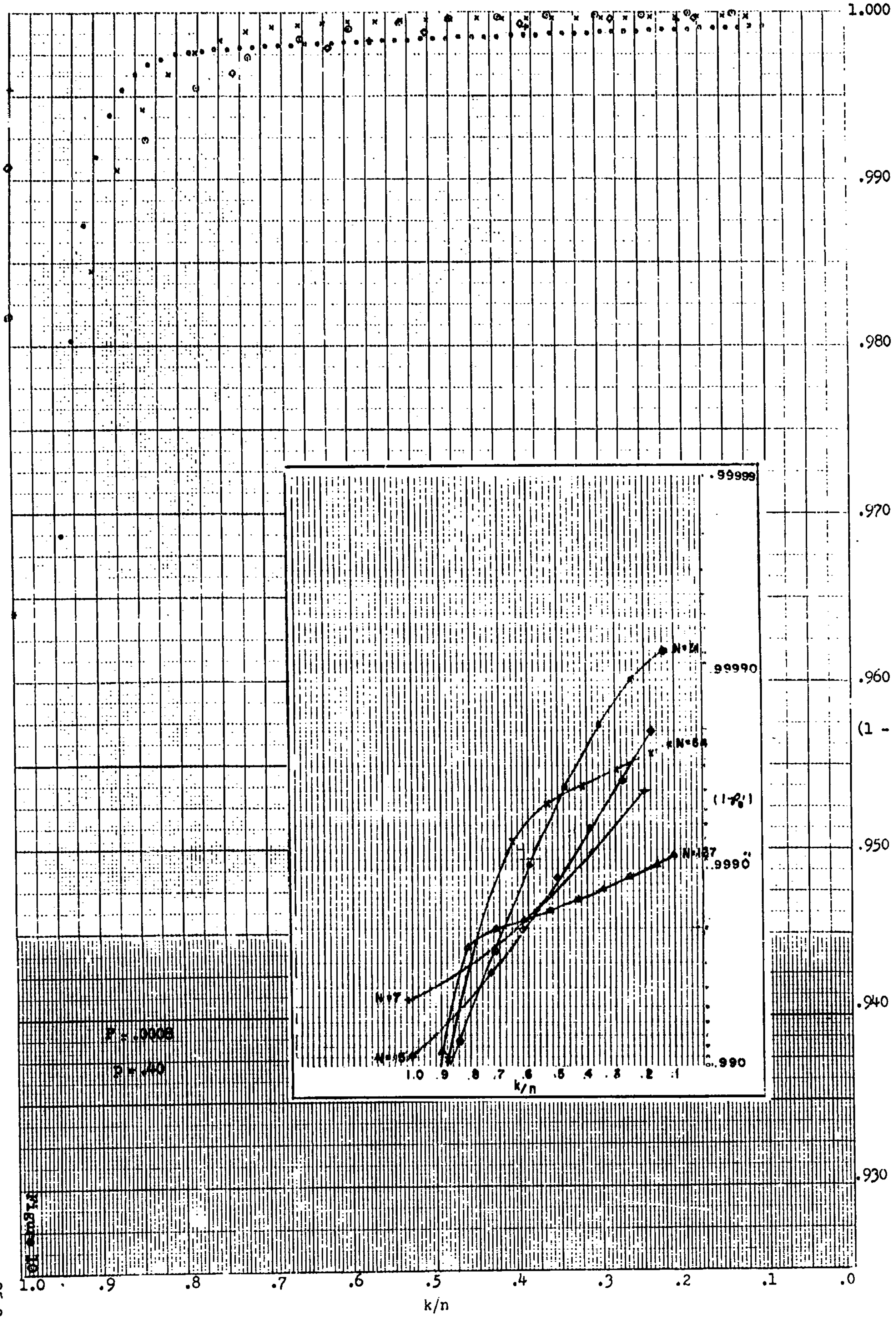
.920
.910
.900
.890
.880
.870
.860
.850
(1 - Pé)
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
P = .0008
p = .20
N = 7
N = 15
N = 31
N = 63
N = 127
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
k/n
.90
.89

Figure 9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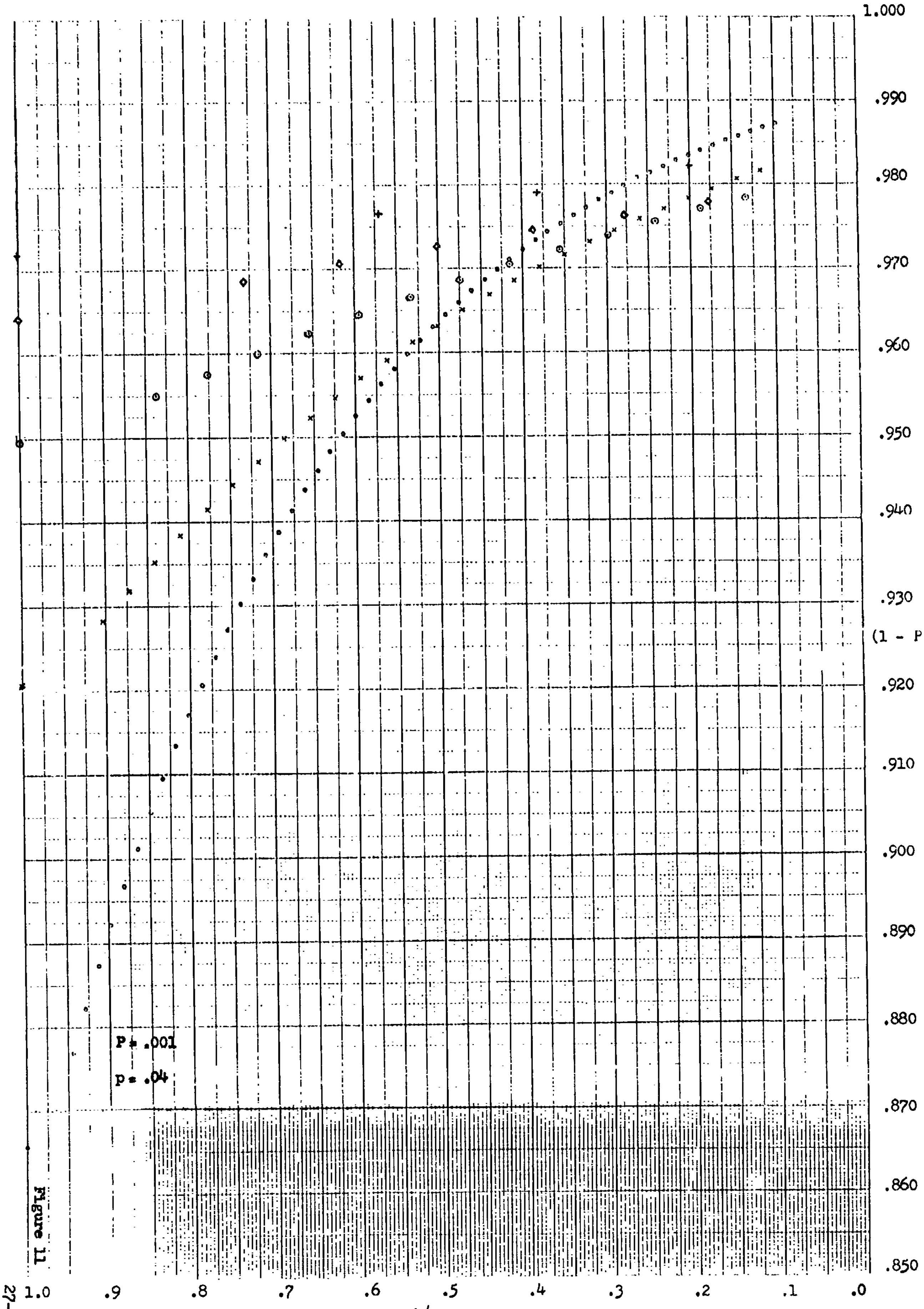
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
.99999
.99990
.9990
.990
N=7
N=15
N=64
N=127
1.0 .9 .8 .7 .6 .5 .4 .3 .2 .1
k/n

Figure 10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.900
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.870
.860
.850
P = .001
p = .04
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.9
.8
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.6
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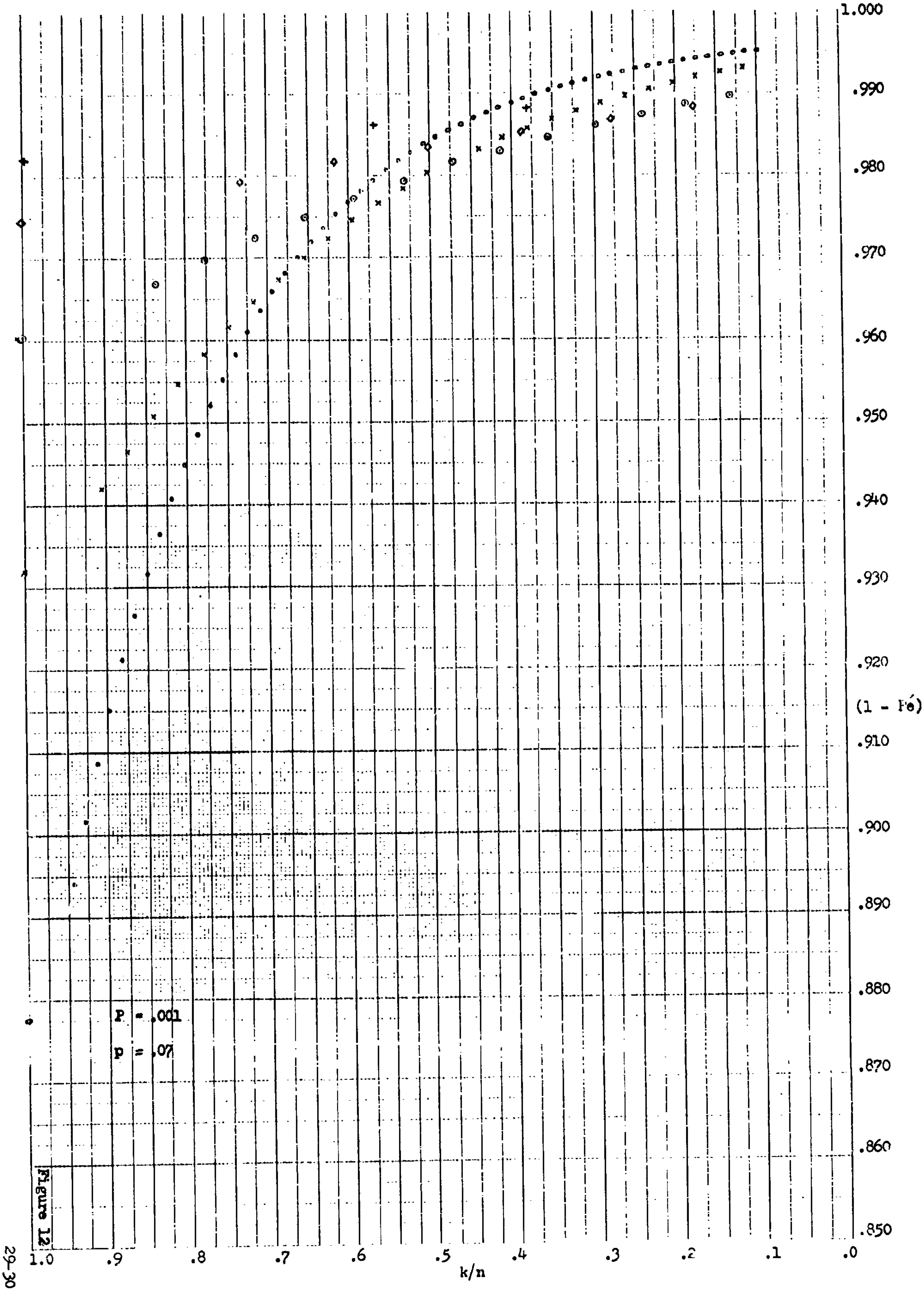
.0
k/n

Figure 11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(1 - Fé)
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.900
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.880
.870
.860
.850
P = .001
p = .07
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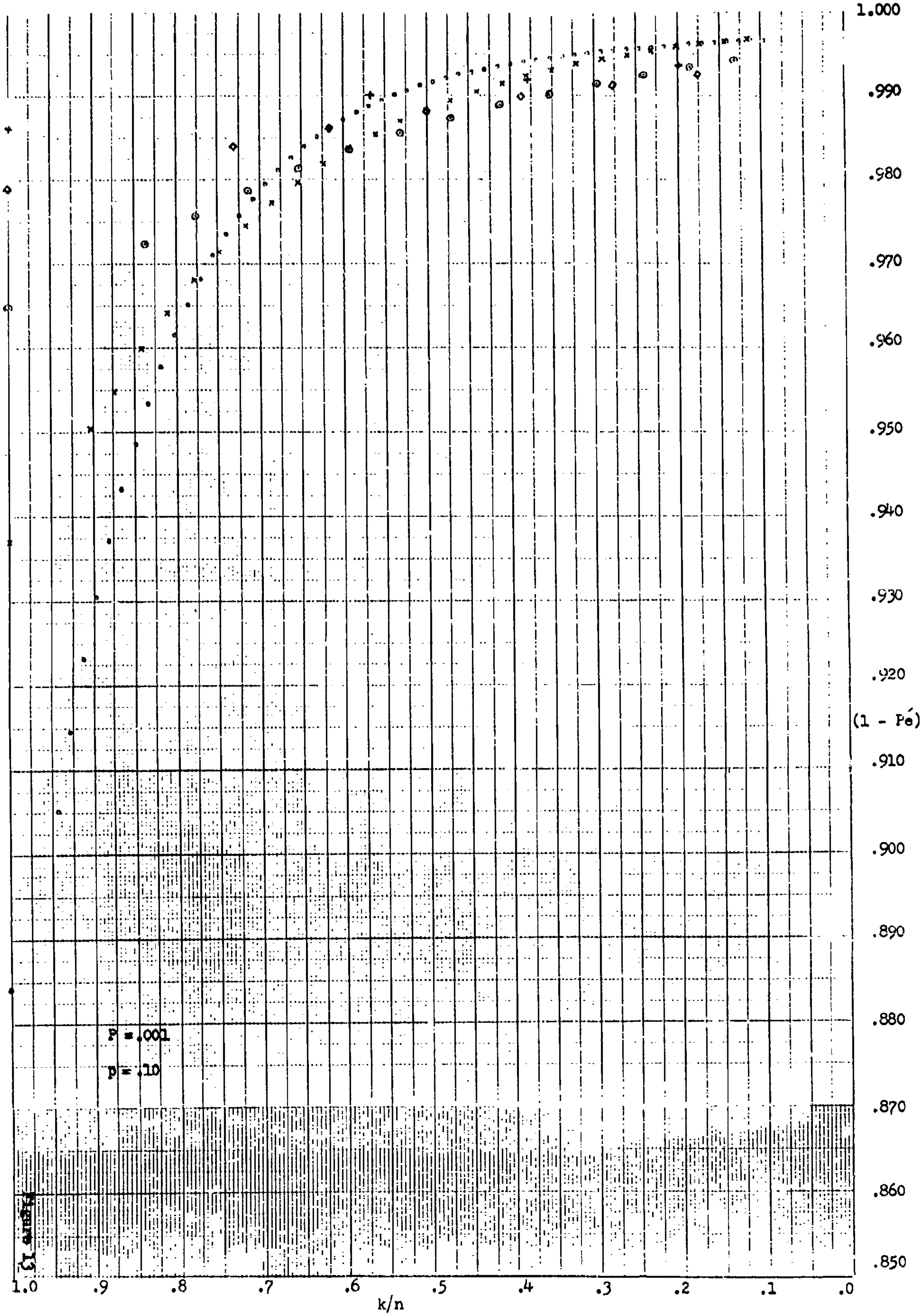
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 12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.930
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(1 - Pé)
.910
.900
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.880
.870
.860
.850
P = .001
p = .10
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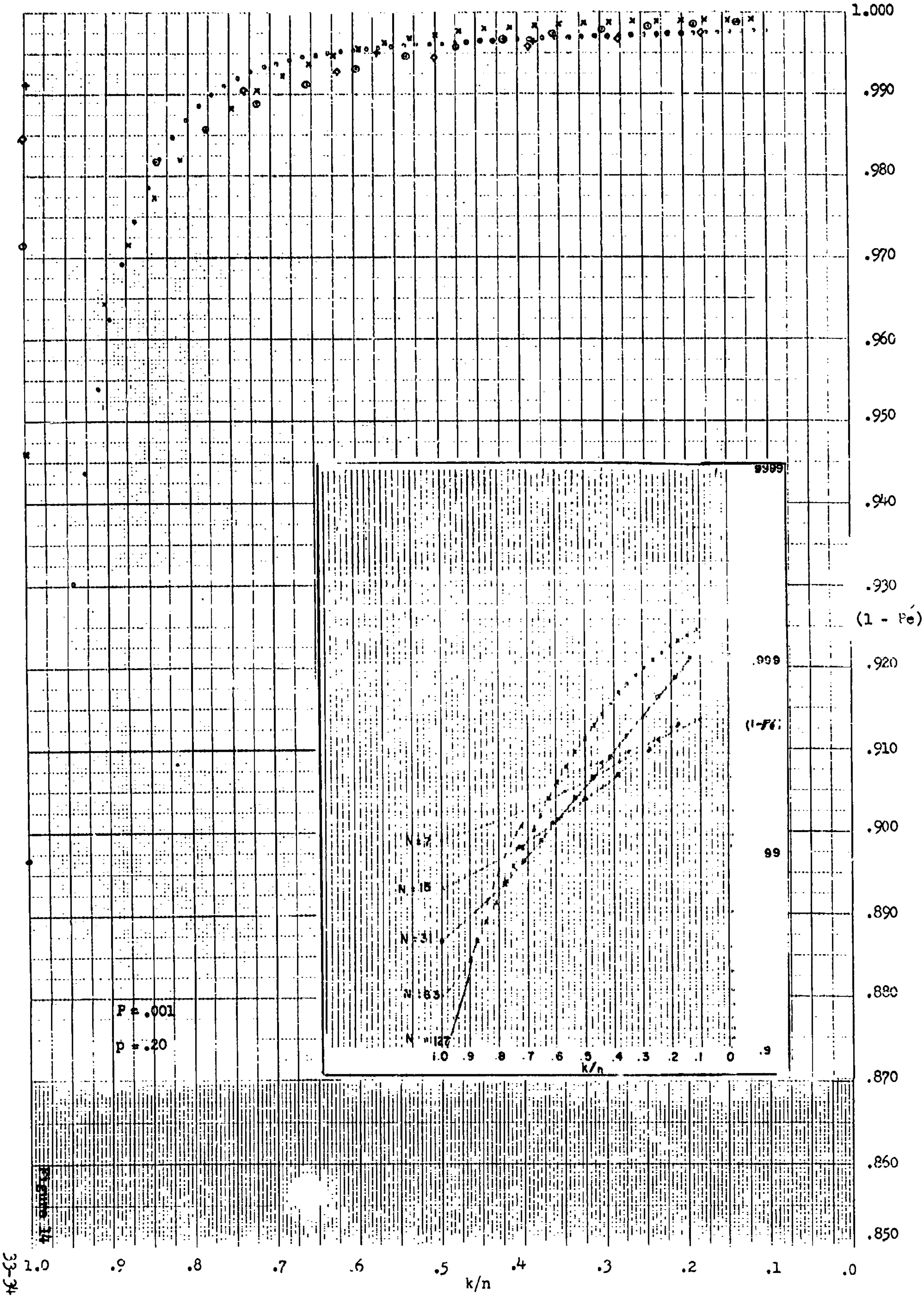
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 13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.870
.860
.850
P = .001
p = .20
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
.9999
.999
(1-Pe′)
99
.9
N = 7
N = 15
N = 31
N = 63
N = 127
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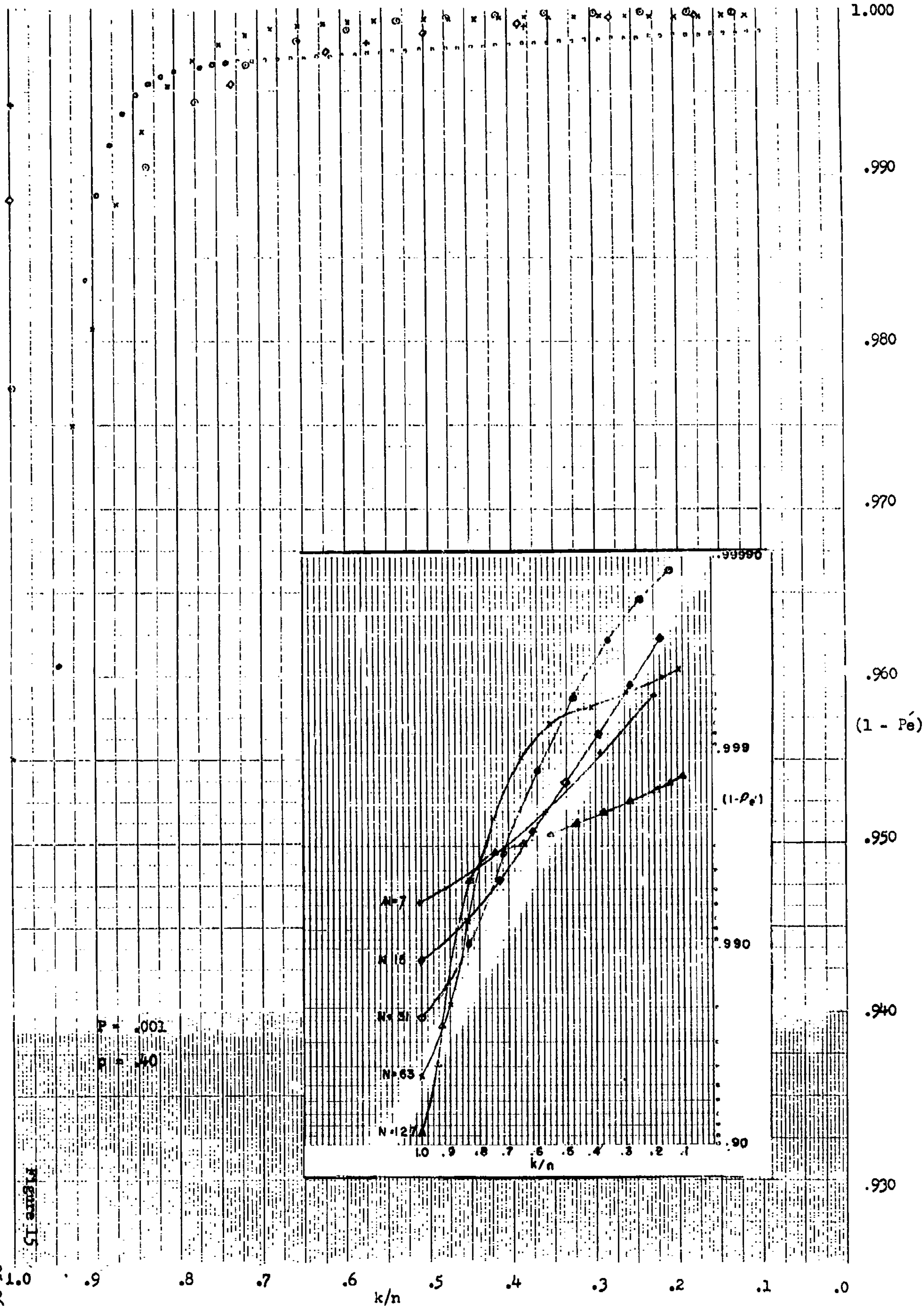
k/n

Figure 14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.7
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.4
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.2
.1
.0
k/n
.99990
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(1-Pe')
.990
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N=7
N 16
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N=127
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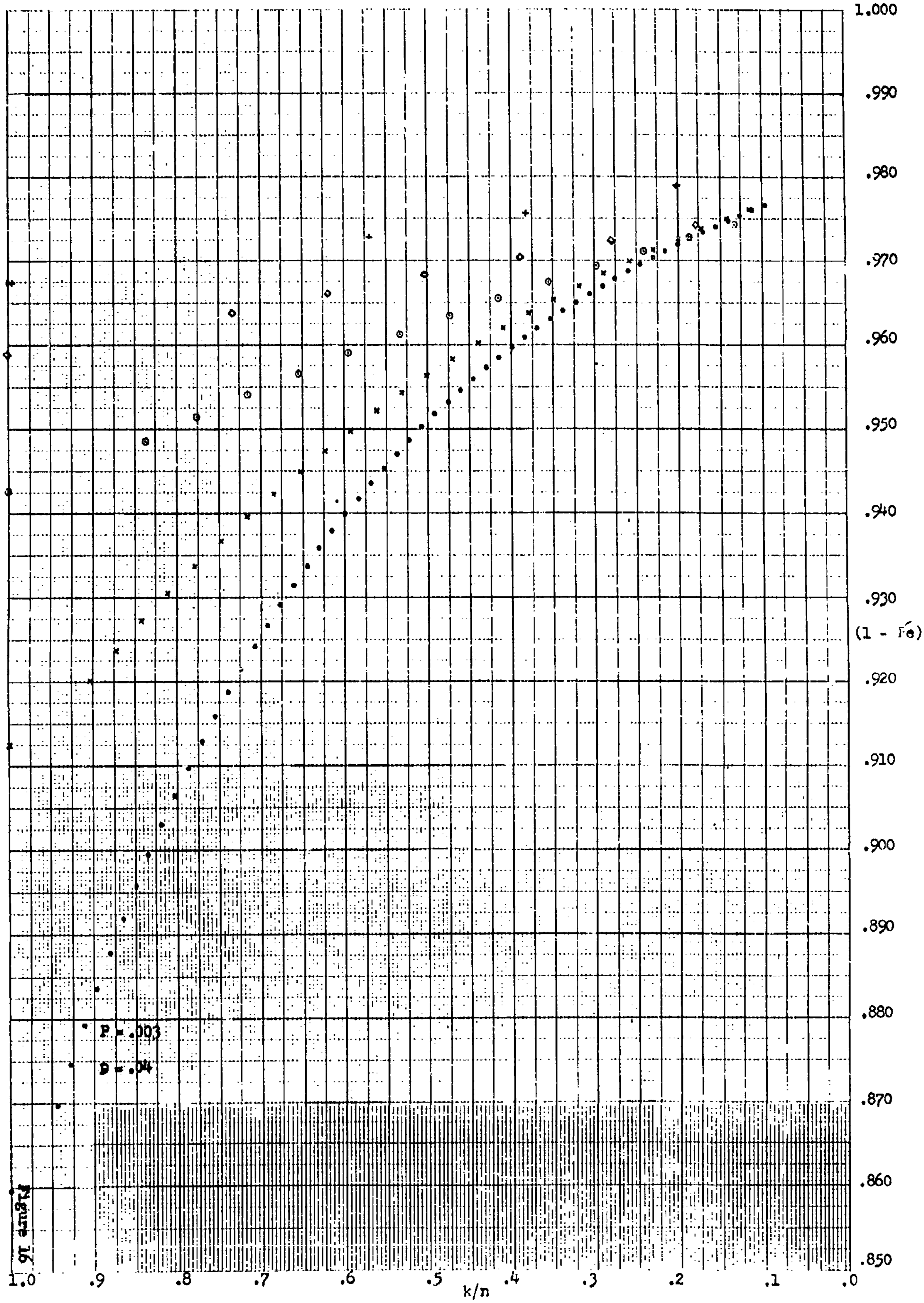
k/n

Figure 15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.860
.850
P = .003
D = .04
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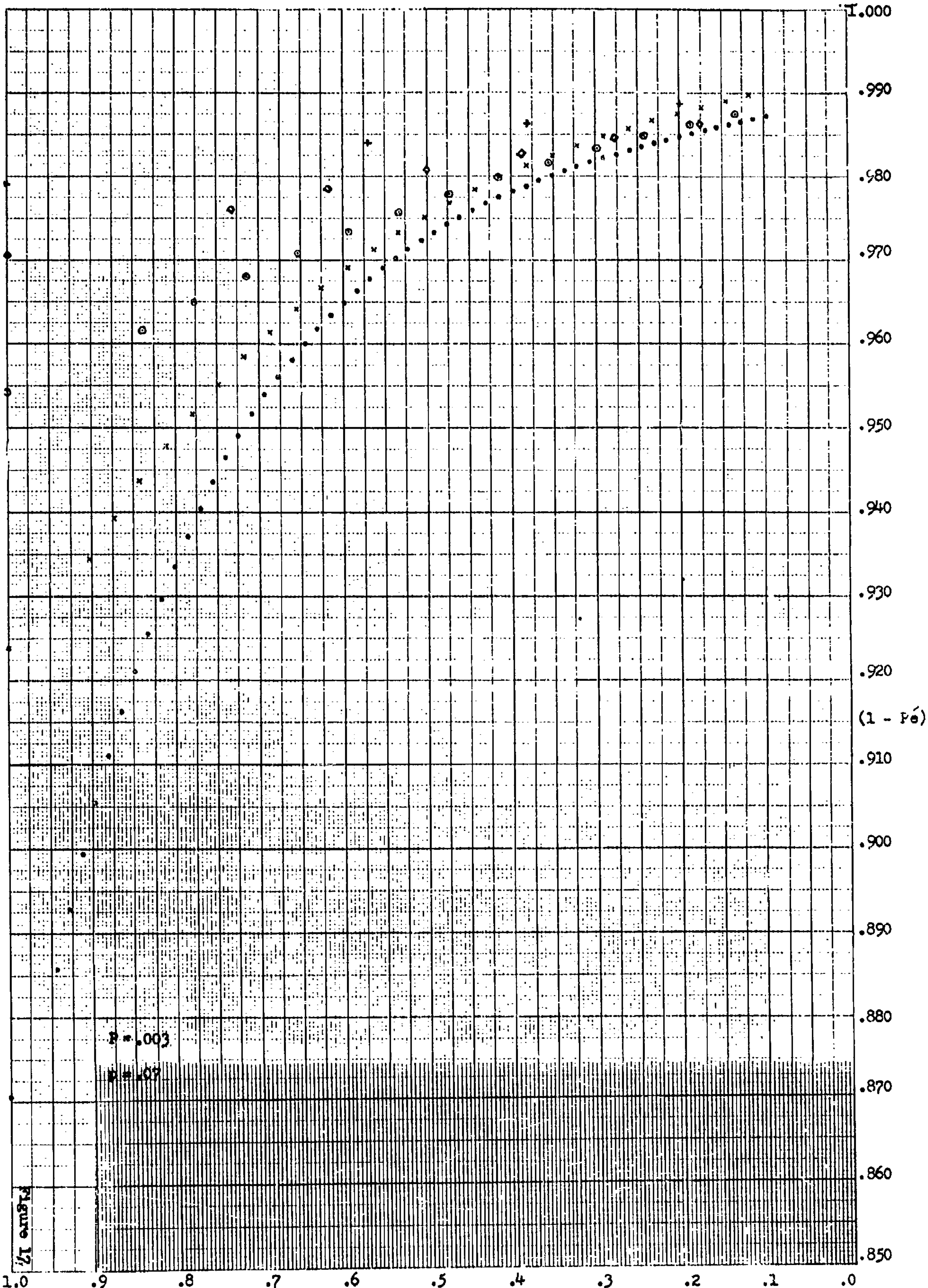
.5
.4
.3
.2
.1
.0
k/n

Figure 16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(1 - Pé)
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.880
P = .003
P = .07
.870
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.2
.1
.0
k/n

Figure 17

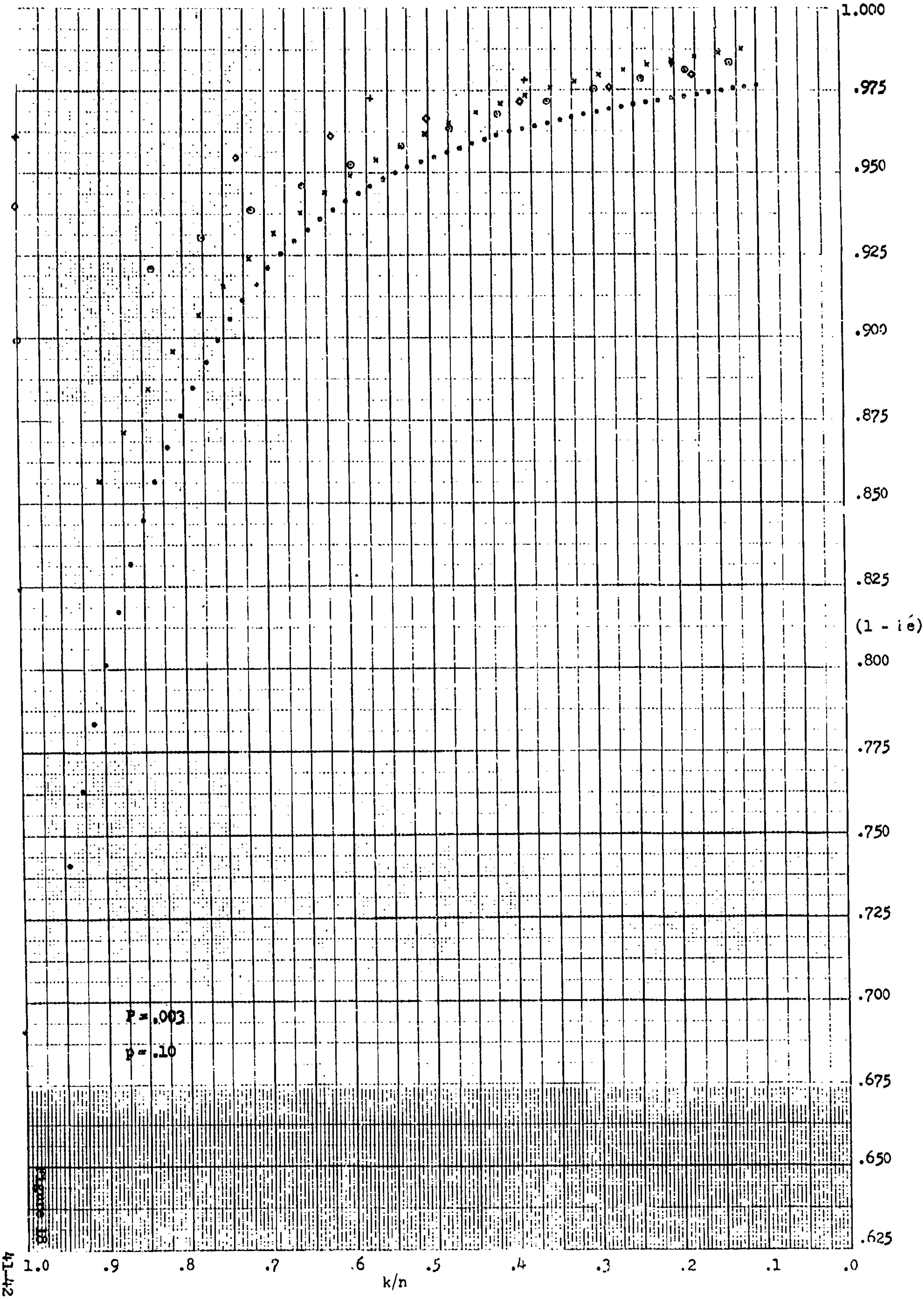
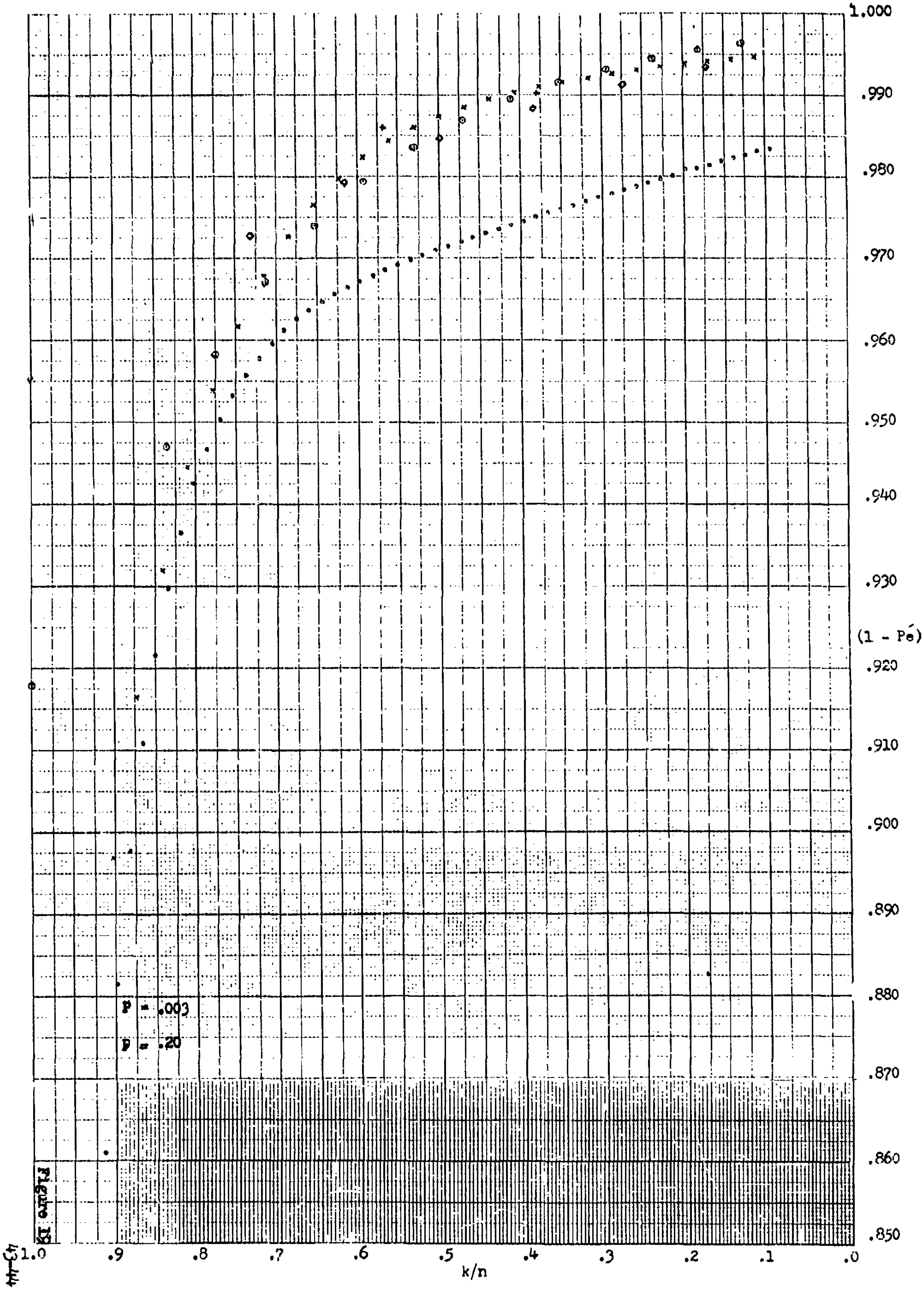
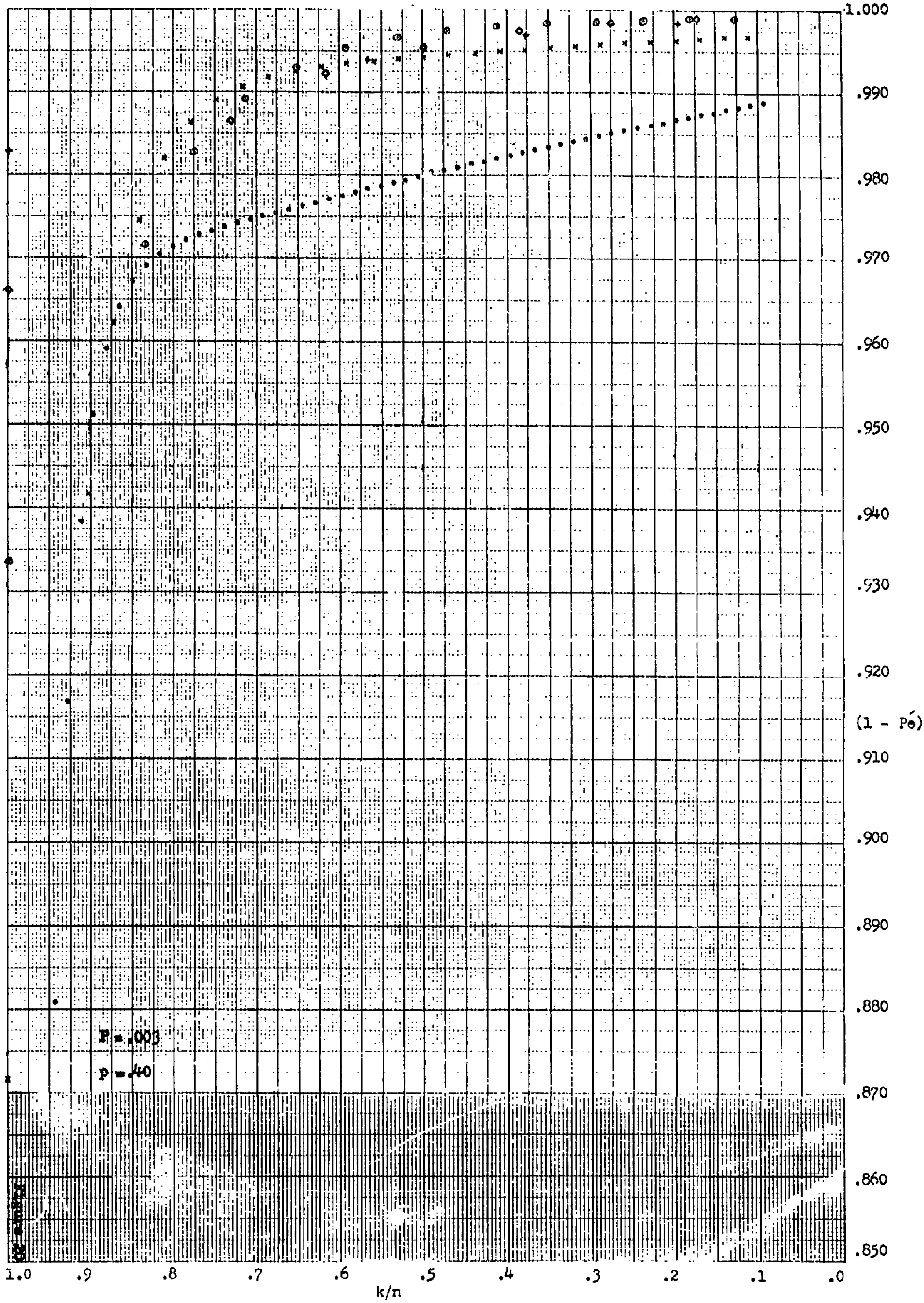

1.000
.975
.950
.925
.900
.875
.850
.825
(1 - i é)
.800
.775
.750
.725
.700
.675
.650
.625
P = .003
p = .10
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 1B



1.000
.990
.980
.970
.960
.950
.940
.930
(1 - Pé)
.920
.910
.900
.890
.880
.870
.860
.850
P = .003
p = .20
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 15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.980
.970
.960
.950
.940
.930
.920
(1 - Pé)
.910
.900
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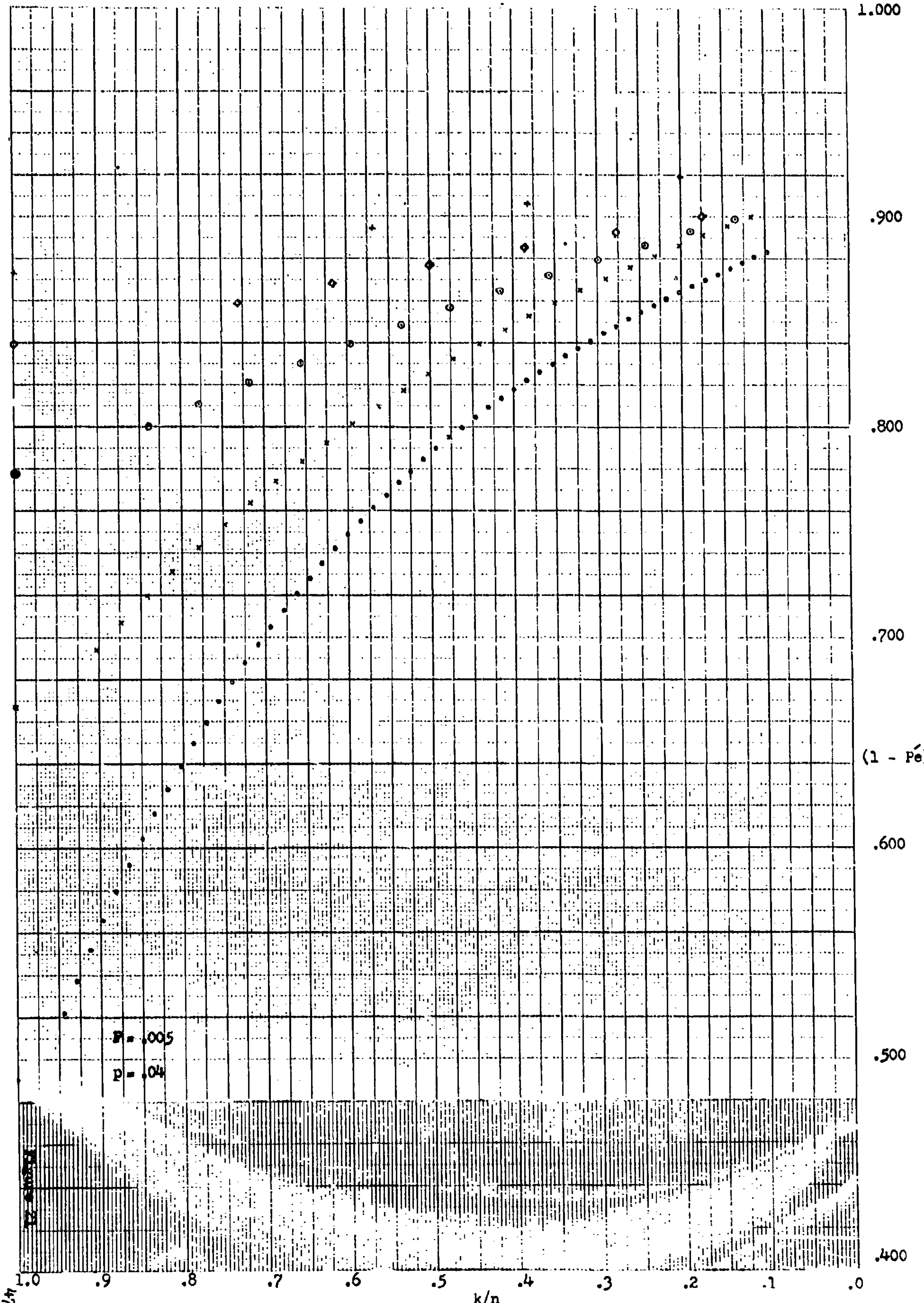
.880
.870
.860
.850
P = .003
p = .40
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n
Figure 20



47-48

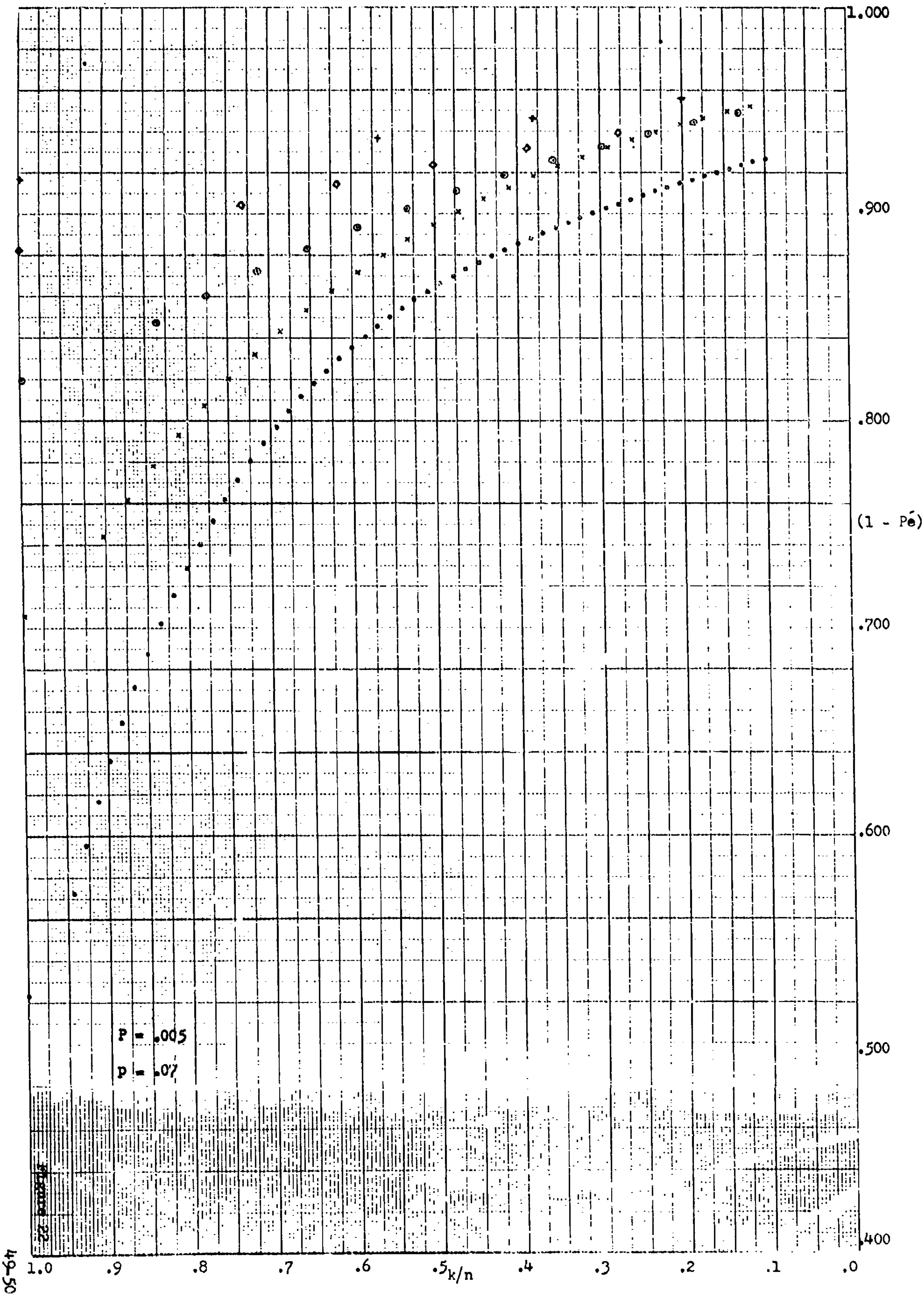


Figure 22

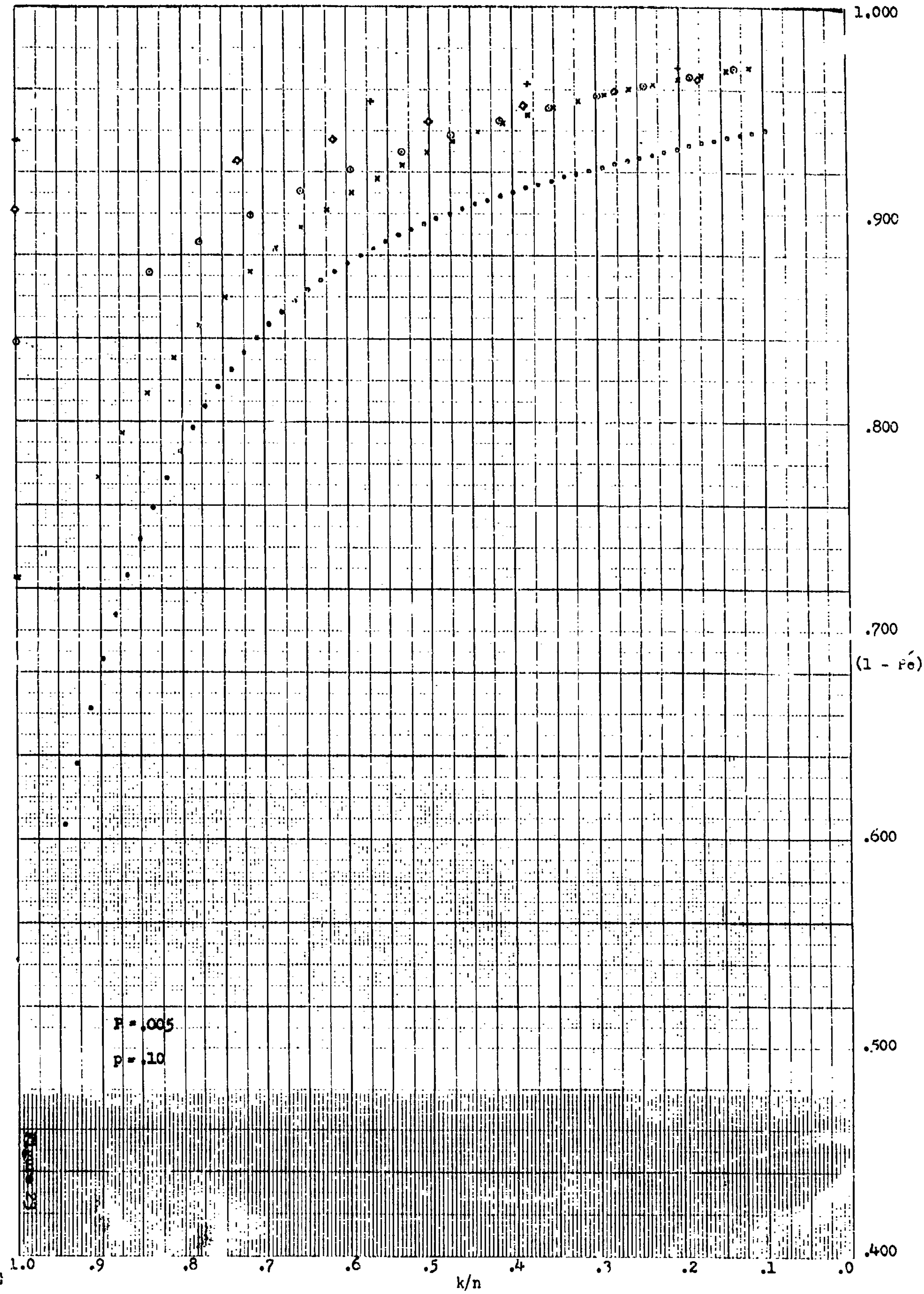


Figure 23

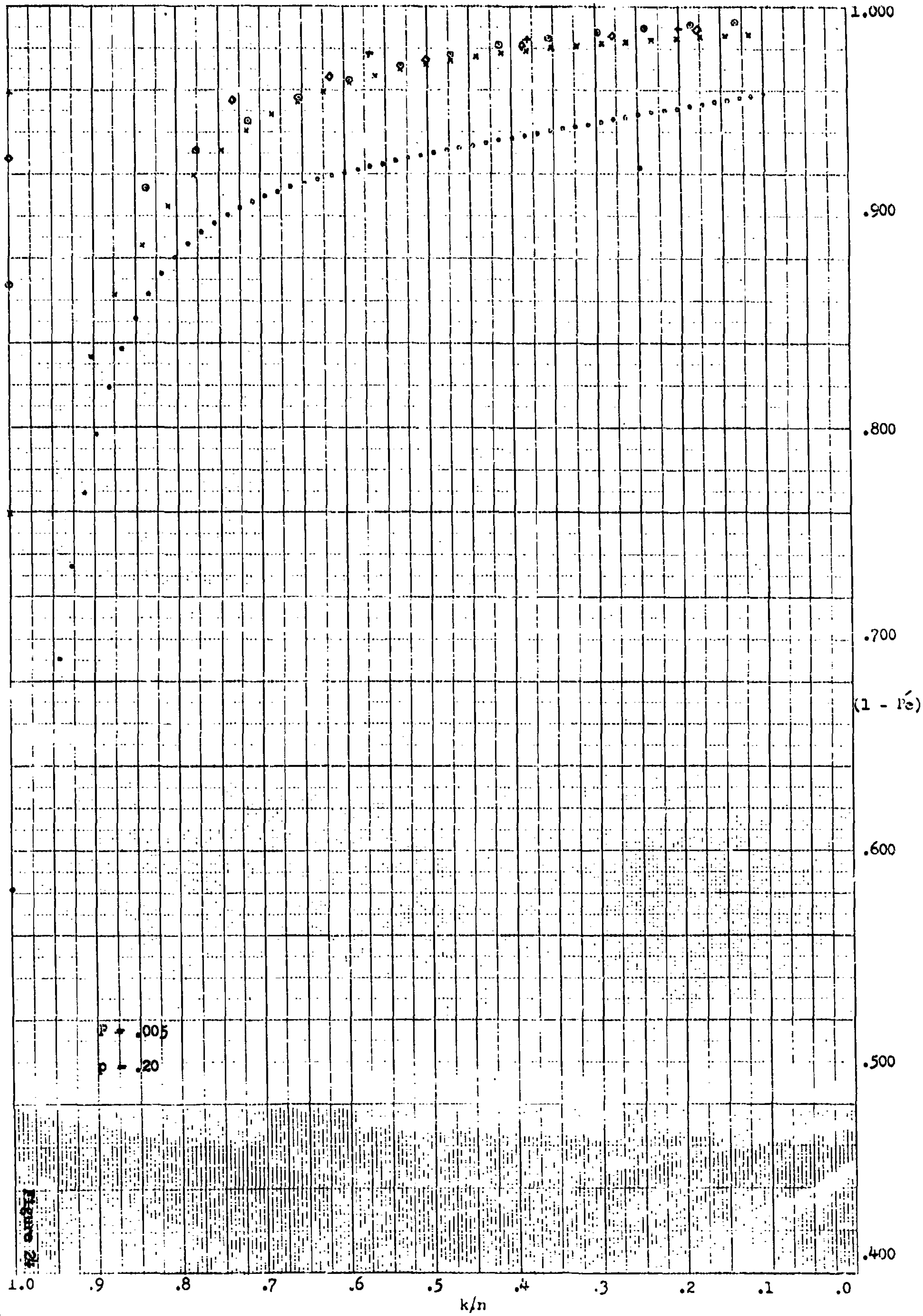


Figure 24

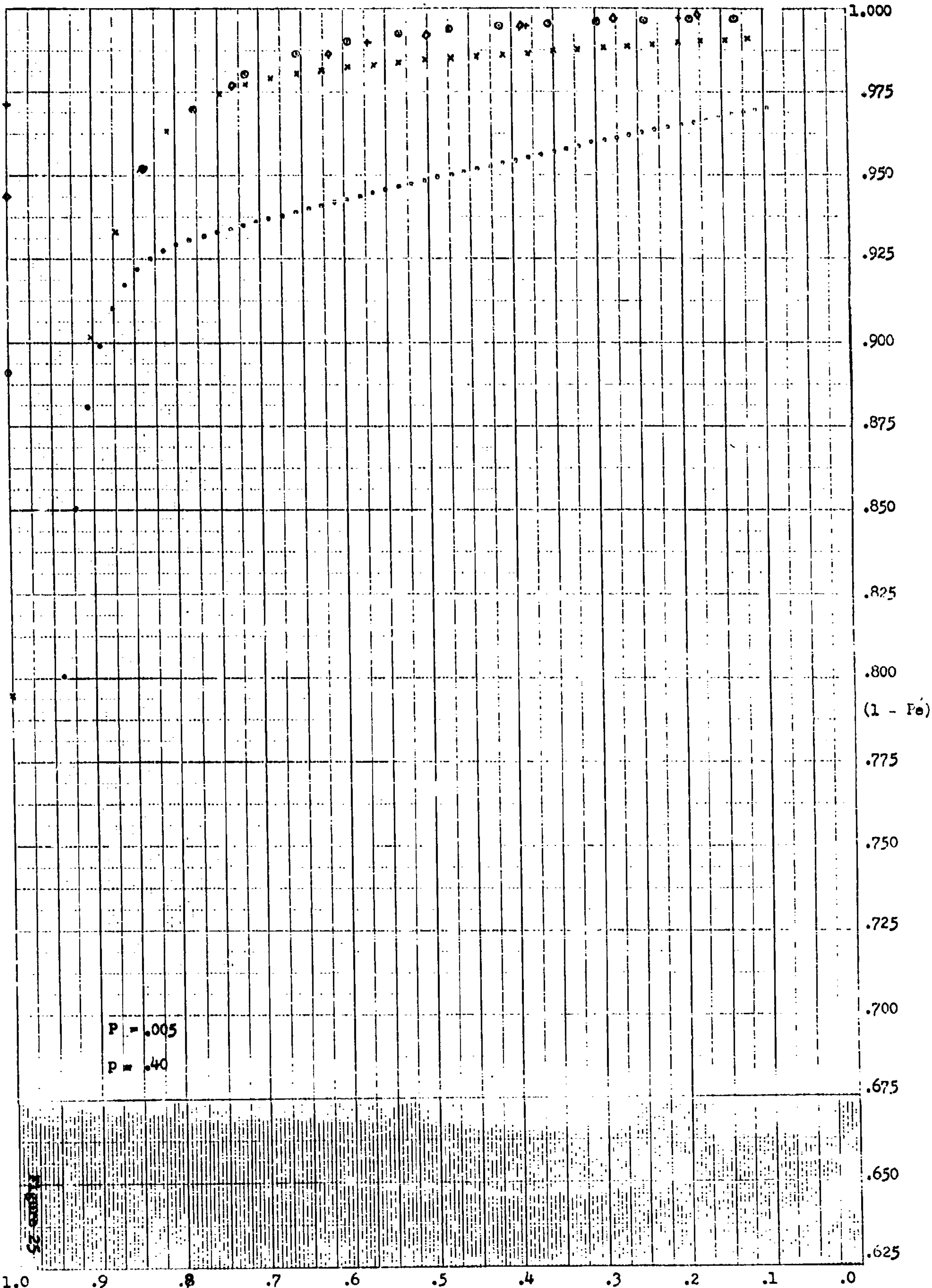

1.000
.975
.950
.925
.900
.875
.850
.825
.800
(1 - Pe)
.775
.750
.725
.700
.675
.650
.625
P = .005
p = .40
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
.0
k/n

Figure 25

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CATALOGUE FILE CARD

Rome Air Development Center, Griffiss AF Base, N.Y.
Rpt No. RADC-TDR-62-189. EVALUATION OF A BURST-ERROR CORRECTION CODE ON A GILBERT CHANNEL. May 62, 57p. incl illus.

Unclassified Report

Error correction codes have been designed to allow the communication engineer to add a degree of error protection to transmitted digital data. Difficulties arise, however, when it is desired to obtain maximum efficiency from the redundant digits added to a particular channel. This report illustrates a technique for evaluating the improvement afforded a fading-type channel by the use of a burst-error correcting code. The Gilbert model of a fading channel is then evaluated using this technique. A series of curves are presented showing the improvement in error rate afforded this particular channel by a burst-error correction code as a function of the reduced information rate.

1. Coding
2. Data Processing
3. Mathematical Analysis
I. Project No. 4519
   Task No. 451903
II. Iram, D. G.
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